Non-measurability of the algebraic sums of sets of real numbers

Consider the following problems:

1. If A is a meagre (null) subset of the real line, does there necessarily exist a set B such that the algebraic sum A+B doesn't have the Baire property (is non-measurable)?

2. If A is a meagre (null) subset of the real line, does there necessarily exist a non-meagre (non-null) additive subgroup, disjoint with some translation of A?

It is not hard to prove, both in the case of measure and category, that the affirmative answer to 2. implies the affirmative answer to 1.

We answer 2. affirmatively for category, while version for measure turns out to be independent of ZFC. The latter fact was essentially proved last year by A. Rosłanowski and S. Shelah. Both results holds for Cantor space with coordinate-wise addition mod. 2 as well.